

Direction Cosine Matrix to Quaternion Conversion

Noel Hughes

noel.h.hughes@gmail.com

Calculating the quaternion equivalent to a direction cosine matrix can be accomplished by generating two intermediate quaternions and multiplying them together. The first of the two intermediate quaternions is that which rotates a unit vector aligned with one of the pre-rotation coordinate axes into the corresponding post-rotation axis. The second intermediate quaternion is generated by rotating one of the other pre-rotation axis vectors using the first quaternion then finding the quaternion that rotates the resultant into the corresponding post-rotation axis. The final step is to multiply the two quaternions together. (vectors are bold, lower case, quaternions are upper case; resultant vectors and quaternions should be normalized after each operation).

Process:

1) define: **dcmx** = first column of the DCM

dcmx = second column of DCM

vx = <1 0 0>

vy = <0 1 0>

2) calculate cross product of **vx** with **dcmx** = **ax1**

if $|\mathbf{ax1}| < \text{small number}$, set **ax1** = <0 1 0>

3) calculate angle, $\text{ang1} = \text{acos}(\mathbf{vx}, \mathbf{dcmx})$

4) form $Q1 = \mathbf{ax1} \sin(\text{ang1}/2) \cos(\text{ang1}/2)$

5) rotate **vy** with Q1

$\mathbf{vy}_r = Q1 \otimes \mathbf{vy}_a \otimes Q1^*$

where

\otimes the quaternion multiplication operator

vy_a **dcmx** with zero appended for compatibility with quaternion multiplication

7) calculate angle, $\text{ang2} = \text{acos}(\mathbf{vy}_r, \mathbf{dcmx})$

8) calculate cross product of **vy_r** and **dcmx** = **tvec**

9) calculate dot product of **tvec** and **dcmx** = **tval**

10) form $Q2 = \text{sign}(\text{tval}) \langle 1 \ 0 \ 0 \rangle \sin(\text{ang2}/2) \cos(\text{ang2}/2)$

8) final quaternion: $Q_f = Q1 \otimes Q2$

Example:

(this is the example used in “Quaternion to Direction Cosine Matrix Conversion” and my paper “Quaternion to Euler Angle Conversion for Arbitrary Rotation Sequence Using Geometric Methods”, references herein are to this paper)

DCM =

0.306185853	-0.250000803	0.918557021
0.8838825	0.433011621	-0.176776249
-0.35355216	0.866024084	0.353553866

First quaternion:

Crossing $\mathbf{v}_x = [1 \ 0 \ 0]$ with the first column of the DCM provides the rotation axis; \cos^{-1} of the (1,1) element of the DCM provides the rotation angle.

$$\mathbf{ax1} = \frac{0.0 \ 0.35355216 \ 0.8838825}{\sqrt{0.0^2 + 0.35355216^2 + 0.8838825^2}} = 0.0 \ 0.37138992 \ 0.928477$$

$$an1 = \cos^{-1}(0.306185853) = 72.17047811 \text{ deg}$$

$$\begin{aligned} Q_1 &= (0.0 \ 0.37138992 \ 0.928477) \sin(72.17047811 / 2) \ \cos(72.17047811 / 2) \\ &= 0.00 \ 0.21874427 \ 0.546861973 \ 0.80814165 \end{aligned}$$

Rotate vector $\mathbf{v}_y = [0 \ 1 \ 0]$ by this quaternion using eqn (15)

$$V_{rot} = Q R V = Q \otimes V \otimes Q^*$$

$$\mathbf{vyr} = [-0.883883874 \ 0.401883965 \ 0.239245846]$$

Normalizing and calculating the dot product of \mathbf{vyr} with DCM(1:3,2) (\mathbf{dcmy}) and taking the inverse cosine produces the rotation angle for Q_2 :

$$\begin{aligned} &[-0.883883874 \ 0.401883965 \ 0.239245846] \cdot [-0.25 \ 0.433011621 \ 0.866024084] \\ &= 0.60218477 \end{aligned}$$

$$ang2 = \cos^{-1}(0.60218477) = 52.97346897 \text{ deg}$$

$$\mathbf{tvec} = \text{cross}(\mathbf{vyr}, \mathbf{dcmy}) = [.3061 \ .8839 \ -.3536]$$

$$tval = \text{dot}(\mathbf{tvec}, \mathbf{dcmx}) = 1.0$$

Thus,

$$\begin{aligned} Q_2 &= \text{sign}(tval)(1 \ 0 \ 0) \sin(52.97346897 / 2) \ \cos(52.97346897 / 2) \\ &= 0.4459906 \ 0 \ 0 \ 0.895037644 \end{aligned}$$

The final quaternion is calculated by multiplying the two intermediate quaternions, or

$$\begin{aligned} Q_f &= Q_1 \otimes Q_2 \\ &= (0.00 \ 0.21874427 \ 0.546861973 \ 0.80814165) \otimes (0.4459906 \ 0.00 \ 0.00 \ 0.895037644) \\ &= 0.360423579 \ 0.439679655 \ 0.391904165 \ 0.723317199 \end{aligned}$$

Which agrees with the original quaternion from the example referenced.